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The application of dynamic reactions of ties in the connection of elements to estimate the capabilities of vibration protection system

S. V. Eliseev[1], A. V. Eliseev[2], A. S. Mironov[3], Q. T. Vuong[4]
[1]Doctor of Technical Sciences, Professor, Chief Researcher - Director of the scientific educational center of modern technologies, system analysis and modeling ISTU, Irkutsk State Transport University. Russia, Irkutsk. ORCID: 0000-0001-6876-8786. E-mail: eliseev_s@inbox.ru
[2]Senior Researcher, Candidate of Technical Sciences, The scientific educational center of modern technologies, system analysis and modeling ISTU, Irkutsk State Transport University. Russia, Irkutsk. ORCID: 0000-0003-0222-2507. E-mail: eavsh@ya.ru
[3]Candidate of Scientific Degree, Scientific and Educational Center of Modern Technologies, System Analysis and Modeling of ISTU, Irkutsk State Transport University. Russia, Irkutsk. E-mail: art.s.mironov@mail.ru
[4]Postgraduate of Scientific and Educational Center of Modern technologies, systems analysis and modeling, Irkutsk State Transport University. Russia, Irkutsk. ORCID: 0000-0003-3026-5301. E-mail: trucvq1990@gmail.com

Abstract. Assessment of the level of vibration effects on the elements of technological and transportation vehicles is one of the most important approaches in ensuring their reliability and operational safety. The aim of the research is to develop a method for constructing mathematical models of technical objects, whose vibrational interactions are estimated by the dynamic responses of the connections of the elements between themselves and the supporting surfaces. The work uses approaches based on the development of methods of structural mathematical modeling and introduces the concepts of dynamic responses of characteristic points of mechanical oscillatory systems, transfer functions of constraint responses and their frequency characteristics. The research demonstrates features of the dynamic properties of systems and the possibility of the occurrence of new dynamic effects. As additional constraints, a motion transformation device is introduced. It is shown that the ratio of the dynamic responses of constraints on the object of protection and on the support surface can be chosen as a parameter of the dynamic state of the system. Analytical dependencies are obtained to determine the coefficient of response dynamism.

Numerical modeling is performed within the scope of the model problem; Dynamic effects are revealed that reflects the properties of the system in the possibilities of creating zones of suppression of external influences when varying adjusting parameters. The possibilities of implementing such approaches are shown through a change in the ratio of spring stiffness in the "cascade".

Keywords: dynamic responses of constraints, transfer functions, motion transformation devices, dynamic stiffness.

Introduction. The reliability and safety of the operation of many technical facilities operating under dynamic loading conditions depends on the level and forms of the occurring vibrations. The management of the dynamic states of technical objects is ensured by special tools and devices introduced into the structure of systems to make it possible to keep the emerging dynamic processes within certain limits. The search of a method and means of vibration protection as one of the urgent problems of modern mechanical science was reflected in the works of domestic and foreign specialists [1, 2]. Reducing the vibrational background of technological and transportation vehicles requires attention to the assessment of the dynamic capabilities of technical objects at all stages of their life cycle, in particular, in predevelopment studies and preliminary calculations [3, 4].

The use of computational schemes of technical objects, followed by the unification of methods, approaches and methods for estimating dynamic states, has become very popular in solving dynamic problems. In this direction, various methods of constructing mathematical models and technologies for their transformation have been developed and are being applied that allow taking into account the features of constructive and technical forms of objects, the conditions for the formation of dynamic states under the influence of various external disturbances, etc.

The methods of structural mathematical modeling [5 ÷ 7] possess certain advantages in the estimation of dynamic effects when mechanical oscillatory systems are introduced as computational schemes of technical objects. Within the framework of this approach, a structural mathematical model in the form of a structural diagram of a dynamically equivalent automatic control system is compared to a mechanical oscillatory system with several degrees of freedom [7-9]. The further use of the analytical tools of the automatic control theory provides the possibilities of applying advanced technologies for frequency analysis of the dynamic properties of systems.

The proposed article develops a methodological basis for estimating the dynamic properties of mechanical oscillatory systems as physical models of technical objects under the influence of periodic external disturbances; a new approach is proposed for estimating dynamic states. It is based on the introduction of such parameters as dynamic responses of the constraints of interacting elements, which requires certain developments in the techniques of structural transformations of mathematical models and appropriate methods for estimating dynamic states.

I. Some general provisions

Many technical objects, in particular, traction electric motors of vehicles (locomotives) are considered as systems with two degrees of freedom, consisting of a solid body making plane oscillatory movements. External influences in such problems are determined by periodic motions of the support surface and are supposed to be known. A schematic diagram of a technical object of this kind (in particular, the traction motor of a locomotive) with a computational scheme in the form of a mechanical oscillatory system with two degrees of freedom is shown in Fig. 1.

The motion of the system is considered in the system of coordinates y_1, y_2, connected with the fixed basis. The system uses elastic elements with stiffnesses k_1, k_2, k_3 and additional constraints in the form of the motion transformation device (MTD). A technical object in the form of a solid body that performs vertical oscillations on elastic supports can be represented, as shown in Fig. 1, by a mechanical oscillatory system.

The system consists of two elastic branches: one is determined by the serial connection of the elastic element k_1 and the block of parallel running springs k_2, and the motion transformation device with reduced mass L.

The characteristic points of connection of the three elements of the branch are pp. (B), (B_1), (B_2). The second elastic branch is represented by a spring with a stiffness coefficient k_3 with characteristic attachment points p. (A) and p. (A_1). The support surface performs harmonic vibrations *z(t)*. The object has mass m; the motion transformation device (MTD) in this case is implemented by a screw non-locking mechanism with a flywheel nut of mass L; the value of this reduced mass depends on the MTD parameters

$$L = \frac{J}{r_{cp}^2 \operatorname{tg}^2 \alpha}. \quad (1)$$

where J is the moment of inertia of the flywheel nut, r_{cp} is the average radius of the thread, and α is the spiral angle of inclination [9].

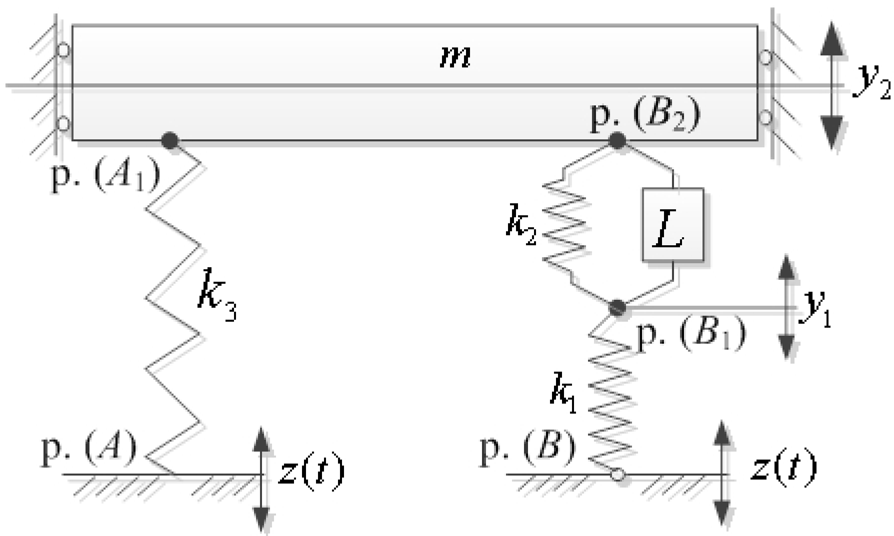


Fig. 1. The computational scheme of a technical object in the form of a mechanical oscillatory system (pp. (A), (A_1), .(B) ÷ (B_2) are characteristic points in which dynamic responses of constraints occur)

The motion of the system is considered in the coordinates y_1 and y_2: y_2 determines the position of the object m, and the coordinate y_2 determines the position of p. (B_1), in which there is a connection of three typical elements of the system (springs with stiffnesses k_1 and k_2 with an MTD having a reduced mass L). It is assumed that the system has linear properties and oscillates with respect to the position of static equilibrium. The coordinate system is connected with a fixed basis, the resistance forces are supposed to be vanishingly small.

1. Mathematical model of the technical object in Fig. 1 can be represented as a system of ordinary differential equations of the second order with constant coefficients. Using the technique given in [7], we find expressions for the kinetic and potential energies of the system in the coordinates y_1, y_2:

$$\mathrm{T} = \frac{1}{2} m\dot{y}_2^2 + \frac{1}{2} L(\dot{y}_2 - \dot{y}_1)^2, \qquad (2)$$

$$\Pi = \frac{1}{2} k_1 (y_1 - z)^2 + \frac{1}{2} k_2 (y_2 - y_1)^2 + \frac{1}{2} k_3 (y_2 - z)^2. \qquad (3)$$

We perform auxiliary calculations and write the equations in the coordinates y_1, y_2 in the time domain:

$$\ddot{y}_1 L + y_1(k_1 + k_2) - \ddot{y}_2 L - y_2 k_2 = k_1 z, \qquad (4)$$

$$\ddot{y}_2 (m + L) + y_2(k_2 + k_3) - \ddot{y}_1 L - y_1 k_2 = k_3 z. \qquad (5)$$

After Laplace transformations under zero initial conditions, the system of equations (4), (5) can be represented in the operator form

$$\bar{y}_1[Lp^2 + (k_1 + k_2)] - \bar{y}_2(Lp^2 + k_2) = k_1 \bar{z}, \qquad (6)$$

$$\bar{y}_2[(m + L)p^2 + k_2 + k_3] - \bar{y}_1(Lp^2 + k_2) = k_3 \bar{z}, \qquad (7)$$

where $p = j\omega$ is the complex variable ($j = \sqrt{-1}$); the icon $\langle - \rangle$ above the variable means its Laplace transform [7].

A structural mathematical model in the form of a structural diagram of a dynamically equivalent automatic control system is shown in Fig. 2.

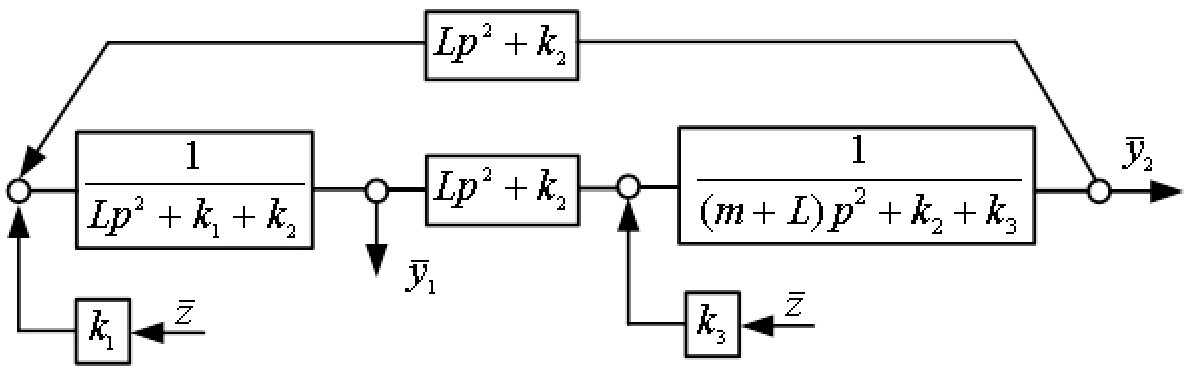


Fig. 2. Structural mathematical model (structural diagram) of the technical object in Fig. 1

From Fig. 2 it follows that the structural model reflects the specific properties of the system; the structure of the system is formed from two partial blocks having elastic-inertial interpartial constraints. At the frequency of the external disturbance

$$\omega^2 = \frac{k_2}{L}, \qquad (8)$$

the interaction between the partial frequencies can be violated. To estimate the features of the dynamic properties of systems with an external harmonic perturbation (in this case this kinematic perturbation *z(t)*) , partial frequencies have a definite value:

$$n_1^2 = \frac{k_1 + k_2}{L}, \ (9) \qquad n_2^2 = \frac{k_2 + k_3}{m + L}, \ (10)$$

which predetermine, in a sense, the possibility of implementing regimes of dynamic damping of oscillations in the system.

2. The transfer functions of the original system in Fig. 1 can be determined from the structural mathematical model or structural diagram in Fig. 2:

$$W_1(p)=\frac{\bar{y}_1}{\bar{z}}=\frac{k_1[(m+L)p^2+k_2+k_3)]+k_3(Lp^2+k_2)}{A(p)}, \quad (11)$$

$$W_2(p)=\frac{\bar{y}_2}{\bar{z}}=\frac{k_3(Lp^2+k_1+k_2)+k_1(Lp^2+k_2)}{A(p)}, \quad (12)$$

where

$$A(p)=(Lp^2+k_1+k_2)[(m+L)p^2+k_2+k_3)]-(Lp^2+k_2)^2 \quad (13)$$

is the characteristic frequency equation of the system.

To evaluate the dynamic responses of constraints, the methodological basis presented in [7] is used, according to which the dynamic response at the characteristic points of the initial system (that is, at the connection points or the contact interaction of its elements) can be found as the product of dynamic stiffness by the magnitude of the dynamic displacement on the coordinate under consideration.

In general, the dynamic stiffness depends on the frequency of the system's oscillations (in this case, on the frequency of the external harmonic kinematic effect). While such approaches are applied to specific schemes, one can usually distinguish the dynamic stiffness of system fragments and dynamic stiffness of individual elements or typical elementary links.

The data on the composition of a set of typical elementary links are presented in [1, 5, 7] with consideration of the elastic, dissipative, inertial properties of elements and motion transformation devices (MTD). In the operator form (Fig. 2) the transfer functions of the elementary links, introduced into the structural mathematical model, have the corresponding form: $W_{elas}(p) = k$ for the usual linear system (k is the spring stiffness); $W_{diss}(p) = bp$ for the dissipative link (viscous friction damper); $W_{iner}(p) = mp^2$ (or Lp^2) for an inertial link or a motion transformation device.

Each of the typical elementary links, in essence, within the scope of structural mathematical modeling, is considered as a link, the input signal in which is the dynamic displacement, and the output signal is the effort (power factor).

In the expressions for the transfer functions of the system (11), (12) the dynamic stiffness will be determined by converting these expressions, which predetermines the representation of the characteristic equation (13), that is, the denominator of the transfer functions (11), (12) in general. If the dynamic stiffness of the system as a whole is zero, this means that under the influence of the harmonic external action a resonance regime will develop, when the motion of the corresponding inertial element, to which an external disturbance is applied, will not face any counteraction movements; the dynamic stiffness of the system as a whole becomes zero at the same time.

3. If the system has two degrees of freedom, then the dynamic stiffness of the system, as a whole, will take zero values twice; such frequencies are frequencies of natural oscillations. The structural diagram in Fig. 2 when considering dynamic responses at characteristic points, for example, at p. (B_2), can be transformed to the form shown in Fig. 3, *a*, *b*. In this case, the element m is an object whose dynamic state is evaluated; the transfer function of an object is interpreted by an integrating link of the second order $\left(\frac{1}{mp^2}\right)$.

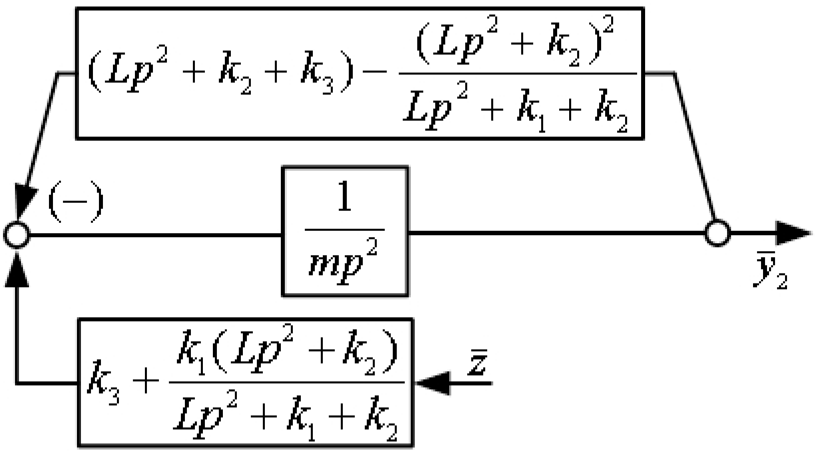


Fig. 3. The structural diagram of the initial system in Fig. 2 with the elimination of the coordinate y_1

In the structural diagram in Fig. 3, the negative feedback loop with respect to the object m represents, in the physical sense, the dynamic stiffness of the structural formation consisting of two branches: the first is a spring with stiffness k_3, the second has the dynamic stiffness of the system fragment of the elements k_1, k_2 and L (cascade), which has been mentioned above.

On the basis of such representations, the dynamic responses of the constraints at the characteristic points of the system (A), (A_1), (B), (B_1), (B_2) can be written as:

$$\left|\bar{R}_A\right| = \left|\bar{R}_{A_1}\right| = k_3 \cdot \bar{y}_2, \quad \left|\bar{R}_B\right| = \left|\bar{R}_{B_1}\right| = k_1 \cdot \bar{y}_1, \quad \left|\bar{R}_{B_2}\right| = \bar{k}_{\text{coer}} \cdot \bar{y}_2 \tag{14}$$

where $\bar{y}_1 = W_1(p)\bar{z}$, $\bar{y}_2 = W_2(p)\bar{z}$, $\bar{k}_{\text{coer}}(p) = k_3 + \dfrac{k_1(Lp^2 + k_2)}{Lp^2 + k_1 + k_2}$.

The dynamic stiffness $\bar{k}_{\text{пр}}$ in the expressions (13) can also be determined as the transfer function of the negative feedback circuit. On the structural diagram (Figure 3) this can be represented as an expression for the reduced dynamic stiffness

$$\bar{k}_{\text{coer}}(p) = \frac{(Lp^2 + k_2 + k_3)(Lp^2 + k_1 + k_2) - (Lp^2 + k_2)^2}{Lp^2 + k_1 + k_2}, \tag{15}$$

which coincides with the previously obtained results in (14). Thus, the dynamic responses characterizing the properties of the suspension can be written as

$$\bar{R}_m = \bar{k}_{\text{coer}} \cdot \bar{y}_2 = \frac{\bar{z}[k_3(Lp^2 + k_1 + k_2) + k_1(Lp^2 + k_2)] \times [(Lp^2 + k_2 + k_3)(Lp^2 + k_1 + k_2) - (Lp^2 + k_2)^2]}{(Lp^2 + k_1 + k_2)A(p)} = \frac{\bar{z}[Lp^2(k_1 + k_3) + k_1k_2 + k_1k_3 + k_2k_3]^2}{(Lp^2 + k_1 + k_2)A(p)}. \tag{16}$$

$$\bar{R}_{\text{supp}} = \bar{R}_A + \bar{R}_B = k_3 \cdot \bar{z} \cdot W_2(p) + k_1 \cdot \bar{z} \cdot W_1(p) = \frac{k_3 \cdot \bar{z} \cdot [k_3(Lp^2 + k_1 + k_2) + k_1(Lp^2 + k_2)] + k_1 \cdot \bar{z} \cdot [k_1[(m+L)p^2 + k_2 + k_3)] + k_3(Lp^2 + k_2)]}{A(p)}. \tag{17}$$

4. To characterize the properties of the suspension, a transfer function of the dynamic links between the responses of the support surface $\left|\bar{R}_{\text{supp}}\right|$ and the responses of the constraints $\left|\bar{R}_m\right|$ created by the external kinematic perturbation $\bar{z}$

$$N(\omega) = \frac{\bar{R}_m}{\bar{R}_{\text{sup}}} = \frac{[-L\omega^2(k_1 + k_3) + k_1k_2 + k_1k_3 + k_2k_3]^2}{\{-[k_3^2 L + k_1^2(m+L) + 2k_1k_3L]\omega^2 + k_3^2(k_1 + k_2) + k_1^2(k_2 + k_3) + 2k_1k_2k_3\} \cdot (-L\omega^2 + k_1 + k_2)}. \tag{18}$$

It follows from analysis (18) that the graph of $N(\omega)$ will have one frequency of "zeroing" the numerator

$$\omega_{\text{dyn}}^2 = \frac{k_1k_2 + k_1k_3 + k_2k_3}{L(k_1 + k_3)}. \tag{19}$$

The denominator of expression (18) can be "nullified" at two frequencies:

$$\omega_{10\text{nat}}'^2 = \frac{k_1 + k_2}{L}, \tag{20}$$ $$\omega_{20\text{nat}}'^2 = \frac{k_3^2(k_1 + k_2) + k_1^2(k_2 + k_3) + 2k_1k_2k_3}{k_3^2 L + k_1^2(m+L) + 2k_1k_3L}. \tag{21}$$

It should be noted that expression (20) coincides with expression (9) for determining the partial frequency.

The introduction of the relation for the dynamic responses $N(\omega)$ at the characteristic points of the suspension (support surface and protection object) has a definite meaning: the kinematic parameters of the motion and the dynamic forces occurring in the joints of the elements should be related to each other in a certain way. When the dynamic loading parameters change, the system should react (or be adjusted) in a certain way. In this case it is proposed to introduce an adjusting parameter in the form of a ratio of the stiffness coefficients of the two suspension branches (k_1 and k_3). The physical possibilities of implementing such approaches in practice exist and are used in active vibration protection systems.

II. Estimation of the dynamic properties of the system

Let us denote

$$k_2 = \beta \cdot k_1,\; k_3 = \gamma \cdot k_1, \qquad (22)$$

then the expressions (11), (12) are respectively transformed

$$W_1(p) = \frac{\bar{y}_1}{\bar{z}} = \frac{k_1[(m+L)p^2 + \beta k_1 + \gamma k_1)] + \gamma k_1(Lp^2 + \beta k_1)}{A_1(p)}, \qquad (23)$$

$$W_2(p) = \frac{\bar{y}_2}{\bar{z}} = \frac{\gamma k_1(Lp^2 + k_1 + \beta k_1) + k_1(Lp^2 + \beta k_1)}{A_1(p)}, \qquad (24)$$

where

$$A_1(p) = (Lp^2 + k_1 + \beta k_1)[(m+L)p^2 + \beta k_1 + \gamma k_1)] - (Lp^2 + \beta k_1)^2. \qquad (25)$$

1. The ratio of dynamic constraint responses represented by expression (18) can be written in the form

$$N(\omega) = \frac{[-L\omega^2(1+\gamma) + k_1(\beta + \gamma + \beta\gamma)]^2}{\{-[m + L(1+\gamma)^2]\omega^2 + k_1[\beta(1+\gamma)^2 + \gamma^2 + \gamma]\} \cdot (-L\omega^2 + k_1 + \beta k_1)}. \qquad (26)$$

From analysis of the transfer function (23) it follows that two resonance regimes are possible with respect to the coordinate $\bar{y}_1$. The frequencies of resonances or natural vibrations are determined by solving the characteristic frequency equation (25). It should be noted that the values of the frequencies of natural oscillations depend on the adjusting parameters β and γ, as well as on the reduced mass L of the motion transformation device (MTD).

With respect to the coordinate $\bar{y}_1$ there can be a mode of dynamic damping of oscillations at a frequency

$$\omega^2_{1\text{dyn}} = \frac{k_1(\beta + \gamma + \gamma\beta)}{m + L(1+\gamma)}. \qquad (27)$$

With respect to the coordinate $\bar{y}_2$ the frequency of dynamic damping of the oscillations is:

$$\omega^2_{2\text{dyn}} = \frac{k_1(\beta + \gamma + \gamma\beta)}{L(1+\gamma)}. \qquad (28)$$

For L = 0, a dynamic damping mode of the oscillations is possible only with respect to the coordinate $\bar{y}_1$.

The ratio of the dynamic responses of the constraints N(ω) can be called the coefficient of response dynamism, since it characterizes the peculiarities of the transfer of force effects from the supporting surface towards the object.

In the general case, N(ω) can have a zero value at the frequency

$$\omega^2_{N\text{dyn}} = \frac{k_1(\beta + \gamma + \beta\gamma)}{L(1+\gamma)}, \qquad (29)$$

which follows from the "zeroing" of the numerator (26).

In turn, N(ω) has infinitely large values at two frequencies

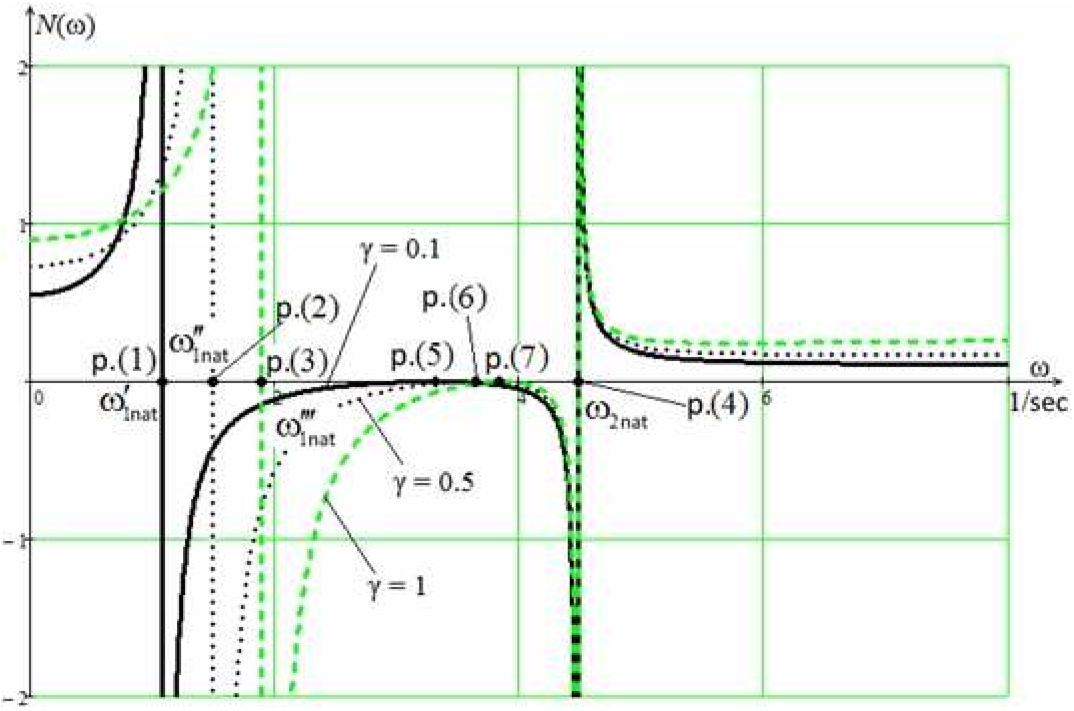
$$\omega^2_{\text{dyn}N_1} = \frac{k_1(1+\beta)}{L}, \quad (30) \qquad \omega^2_{\text{dyn}N_2} = \frac{k_1[\beta(1+\gamma)^2 + \gamma^2 + \gamma]}{m + L(1+\gamma)^2}. \qquad (31)$$

As the frequency of the external influence ($\omega^2 \to \infty$) increases, N(ω) tends to the limiting value

$$\underset{\omega\to\infty}{N(\omega)} = \frac{L(1+\gamma)^2}{m + L(1+\gamma)^2}. \qquad (32)$$

2. Frequency characteristics (FC) of the constraint responses are shown in Fig. 4, *a*, *b*. For example, the following parameters are chosen in the model problem: m = 1000 *kg*, L = 100 *kg*, k_1 = 1000 *N/m*, β = 1.

a)



(b)

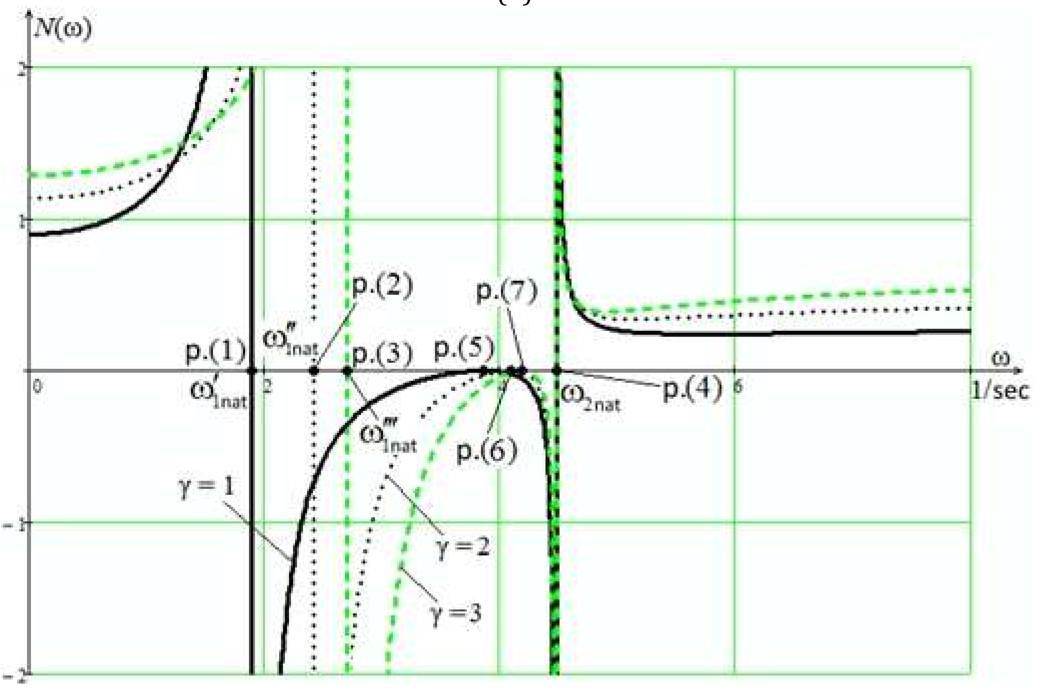


Fig. 4. Frequency characteristics of the responses of constraints at the characteristic points of the mechanical oscillatory system (pp. (A), (A_1), (B), (B_2)):
a) γ = 0.1, 0.5, 1; *b*) γ = 1, 2, 3

In Fig. 4, *a*, *b* pp. (1), (2), (3), (4) reflect the shift to the right (in the direction of increase) of the frequencies of resonance increase $N(\omega)$. The points on the abscissa axis (pp. (1), (2), (3), (4)) reflect the positions of the corresponding frequencies of increase in amplitudes of $N(\omega)$ to large values. When comparing pp. (1) ÷ (4) in Fig. 4, *a* and pp. (1) ÷ (4) in Fig. 4, *b,* there is a shift of frequencies to the right, observed before the coincidence of pp. (4) in Fig. 4, *b*, with increasing the adjusting parameter.

In the limiting cases (Figure 4, *b*) the points can coincide, which indicates a strong influence of adjusting parameters on the distribution of dynamic responses in the interactions of the elements of the system. When comparing the FC for different γ, we note that the frequency of the dynamic damping of the oscillations (pp. (5), (6), (7)) is shifted to the right to p. (4), which is due to the separate dependence of the dynamic states on the adjusting parameters β and γ.

2. Fig. 5 shows the FCs of dynamic responses in isometric form, where the spatial distribution of the graphs of the $N(\omega)$ dependences has an additional coordinate axis along which the adjusting parameter γ is plotted.

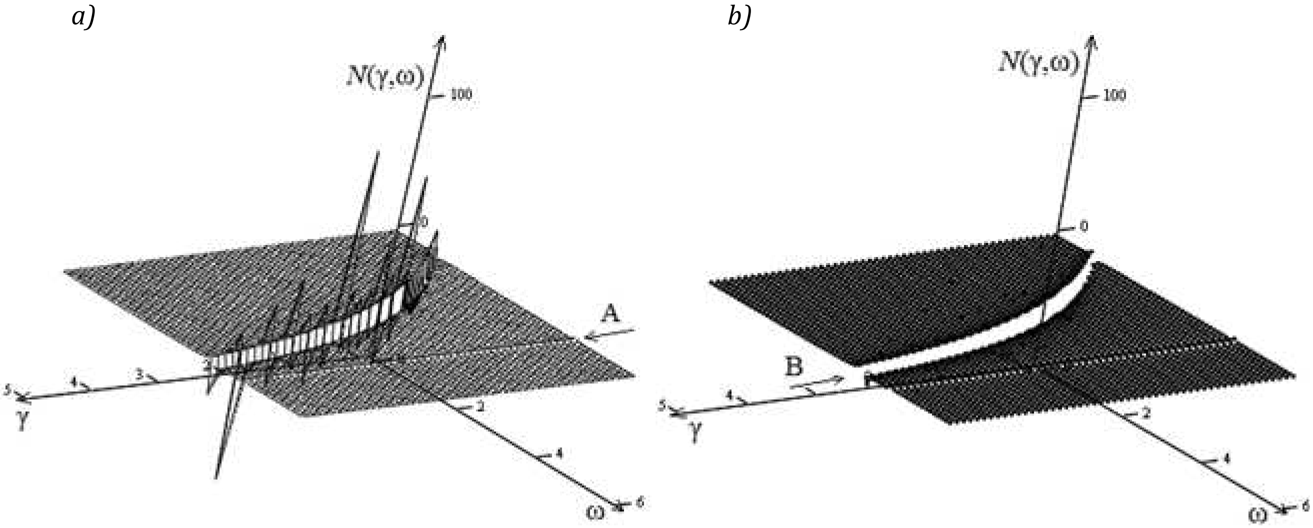


Fig. 5. The graph of the $N(\omega)$ dependences in isometric form: a) surface graph; b) data points

Fig. 5 shows an isometric representation of the frequency-characteristic relations of dynamic responses depending on the frequency of the external influence, taking into account the deep variation of the parameters. Fig. 5, *a* denotes the zone of abrupt change in the values of $N(\omega)$ that occur at frequencies, increasing (or amplifying) the transmission of power disturbances.

The spatial diagram (Figure 5, *a*) presents two regions of the values of the intensive increase in dynamic response; the zone marked by the arrow A (Figure 5, *a*) is shown in detail in Fig. 6, *a*.

Fig. 6, *b* shows the overall picture of changes in the values and forms of the dynamism coefficients, which makes it possible to assess the general conditions for the influence of adjusting parameters on the dynamic properties of the suspension system.

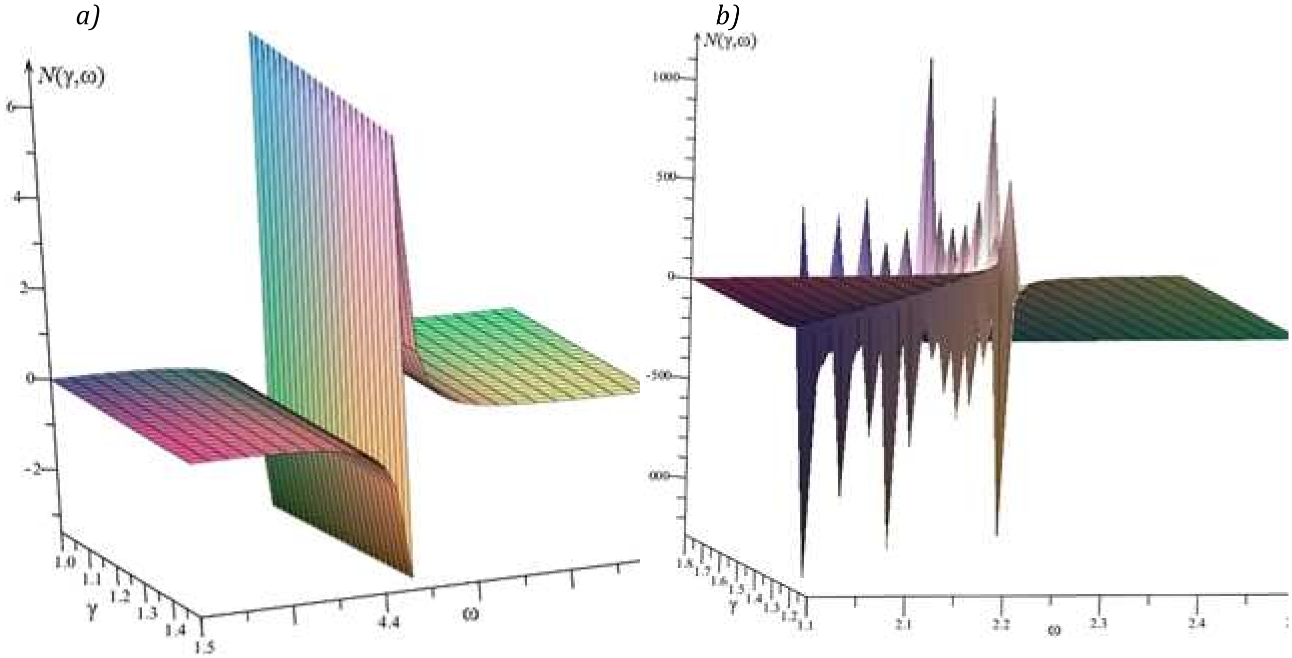


Fig. 6. Detailed view of the graphs in Fig. 5: *a*) the zone marked by the arrow A (Figure 5, *a*); *b*) the zone marked by the arrow B (Figure 5, *b*)

Conclusion

This paper develops new methodological approaches to evaluate the dynamic properties of mechanical oscillatory systems considered as computational schemes of technical objects. It is shown that the traditional ideas about the formation of the dynamic properties of mechanical oscillatory systems, including suspensions and suspension systems of vehicles, based on a comparative analysis of kinematic parameters, can be additionally detailed by taking into account the dynamic responses of constraints.

The work introduces a number of new concepts that reflect the features of dynamic interactions of system elements, which is reflected in the frequency characteristics of the dynamic responses of constraints. Such approaches create the prerequisites for the development of a comprehensive analysis of the overall picture of the interaction of the elements of the system and for the creation of conditions for improving the reliability of the machine components designed with allowance for a more complex system of vibrational loading during intensive machine operation.

In general, the work is related to the development of a new method for estimating the dynamic properties of objects, vibration loadings, as well as the technologies for its implementation, using the example of the analysis of the transport suspension model as a mechanical system with two degrees of freedom.

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